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# TAU ELECTRON ATOMS AT RHIC

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An amusement ancillary to the proposed quark-gluon plasma production hypothesized from a relativistic heavy ion collider (RHIC) may be a sufficient quantity of tau electrons to potentially admit the study of its exotic atoms. Given the wealth of nuclear phenomena derived from muonic atoms one might assume a tau atom would be even more forthcoming of information due to its lower orbits being entirely contained within the nucleus. It is the purpose of this brief note to discuss the production mechanism at a RHIC and to delineate some of the more obvious properties of the tau atom. As in the case of the mu, no doubt a host of more exotic phenomena derived from resonance "accidents" with nuclear transitions will also take place, but it would be presumptious to discuss them at this time. Given the complete containment in nuclear matter of the tau lepton in its innermost atomic orbits, one would think the most immediately interesting, albeit potentially fruitless, experiment to be performed with such an exotic species would be the measurement of its lifetime.

These simple speculations are motiviated by the work of Harvey

Gould[1], who has calculated background processes at a RHIC. Among the plethora of peripheral processes will be the production of lepton pairs by "the photon field" of one nucleus interacting with the coulomb field of the other. (In a collider one cannot use the comforting target and projectile nomenclature and I do not know of a euphonic alternative.) For the counterrotating beams of fully stripped Uranium at 100 Gev/A design that Gould considers, he estimates $10^8$ electron pairs, $2*10^3$ muon pairs and 1 tau pair would be produced per second. These are undoubtedly upper limits in that he ignores finite size effects which clearly will be most important for the tau, with its mass of 1784.2 MeV. A lower bound would reduce his estimate by two orders of magnitude but one can make arguments that that clearly would be a gross exaggeration. In fact the question of realistically incorporating finite size effects and inelasticity is not at all trivial and I am not aware of any such published treatment. Hence with the caveat that what follows may be an amusing exercise in futility unless the luminosity of $10^{27}$ $cm^{-2}$ $s^{-1}$ that Gould uses as a design figure were increased to compensate for the potential finite size decrement, we will proceed to calculate the spectra of a negatively charged tau electron captured by Pb.

However before doing so, some of the relative time scales involved should be discussed lesst we waste time on the energy levels of a quantity whose unproducibility is exacerbated by its emphemerality. With a weak decay liftime of $(3.4\pm0.5)*10^{-13}$ seconds the inherent instability of the tau- is only an order of magnitude longer than the assumed cascade time of a mu- of $10^{-14}$ seconds. However, the mu- spends a great deal of this time losing energy by the auger process[2] at very distant orbits from the well shielded nucleus. The tau-, at a RHIC, is instead created in the field of a bare nucleus which will both facilitate its capture and lead it into interesting x-ray producing orbits more quickly than a mu-. The other process which might vitiate the existance of such exotic tau atoms is inverse beta decay. For a mu- this is bounded by rates of $10^6$ to $10^8$ per second. Taking the latter, appropriate for a heavy nucleus with the mu- half inside the nucleur material, and scaling by phase space and a factor of two for complete immersion we are still three orders of magnitude short of the natural lifetime. If we ignore decreasing phase space as we proceed up the baryon energy scale, this must be multiplied by the number of baryons the proton could be converted into with the caveat that the baryon energy must be less than 3 Gev. Fortunately, this still leaves considerable room for new particles before this rate approaches the weak decay liftime. However, the weak decay liftime will produce a natural lower bound for the line width of the tau- x-rays of approximately 0.1 volts. It is improbable that this limiation will seriously inconvenience experimenters in the 1990s trying to measure, as we will see, Mev x-rays, in a system moving at nearly the velocity of light.

The Bohr radius of a tau atom of lead is approximately 0.1 fm which is picuant but irrelevant given a nucleus 12 or 14 fm in diameter. We solve the non-relativistic Schroedinger equation, with for oriention purposes, a uniform charge distribution with sharp cutoff in radius at 1.1*A1/3, which takes the well known form:

$$-\frac{\hbar^2}{2m}\Delta^2\phi + V(r)\,\phi = \epsilon\phi \qquad \text{where}$$

$$V(r) = \frac{Ze^2}{2R_o}\left(\left(\frac{r}{R_o}\right)^2 - 3\right)\ ;\ r < R_o;\ = \frac{Ze^2}{r}:\ r > R_o$$

For simplicity we have taken the tau mass to be twice a nucleon mass and ignore any spin or spin-orbit effects. From Table I we see the results for the first six orbits for S, P and D states. Displayed are the eigenvalues and radii at which the wave functions have their maximum. One notes that only the lowest states are fully contained in the interior of the nucleus and impervious to the surface. This also manifests itself in Table II where the energies of the non-node changing transitions are calculated. Hbar omega is 2.9 Mev for the sharp cutoff model, with these parameters, and we see an harmonic spectrum only for the first orbits. As nodeless hydrogen orbits scale approximately as $n^2$ where n is the sum of radial quantum number + angular momentum +1, we cannot expect and do not see any hydrogen like spectra by n=6. In fact, beyond n=6, we are very close to the onset of a continuum in x-ray energies, which will include the hydrogen quasi continuum limit but proably not permit its separation from those states affected by the finite nuclear size.

For comparison, but not for realism, we have also calculated the same spectra using a woods-saxon charge distribution with the same rms radius as the sharp cut off model but with two values of the surface diffusion, the conventional 0.65 f and 0.55 f. These results are contained in Table I-Model II, and Table I-Model III, respectively. Table II includes the predicted x rays from all three models.

One sees that the spectrum is nearly harmonic for all three models only for the lowest orbits (1d-1p, 1p-1s) and that the models differ among themselves at the 100 kev level for these transitions. This is clearly the only region insensitive to the nuclear shape or surface and provides access to the true interior of a nucleus (whether or not this would represent nuclear matter is a subject of speculation). The other transitions for the models differ by similar quantities. Given the expected result that some of the spectra will be sensitive to the nuclear shape will no doubt lead to the same type of sophisticated analysis that the muon has seen, if data are ever available. However, the weighting will be very different. Prior to designing an experimental program one would of course engage in the more detailed analysis requisite from coupling atomic and nuclear levels[3] which will shift many of the levels calculated here by at least several percent. It is our purpose only to point out the gross features of the spectrum.

It is clear that the spin-orbit splitting will be quite small due to the large mass of the tau, approximaely $10^{-8}$ of the potential energy which is no more than 30 Mev. Alternatively, for the sharp cut-off potential (Model I), the effect of the spin-orbit force is to add a small linear term to the potential energy for a given j, which displaces the zero of the potential slightly. Again, one sees that the effect is presumably unobservable with present techniques. Finally, examining the energy levels of a tau atom inside or adjacent to the high magnetic field due to the accelerator may offer some possibility for zeeman effect obervations but that is too speculative even for this fantasy.

The calculations offerred here were for Pb even though the presumed RHIC design is to accelerate Uranium. Uranium has its own special cachet when it comes to forming an exotic atom because of its deformation and fissibility, which would obscure the naive calculations presented here. However, its muonic atoms offer some very amusing properties[4] and no doubt refined calculations of its tau atoms will be forthcoming of similarly enriching phenomena.

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## TABLE CAPTIONS

Table I: e is the eigenvalue in Mev in descending order for L=0, 1 and 2. Space is the total radial distance over which numerical integraton was performed. Max Wf is the distance from the origin at which the wave function has its maximum. Delta is the difference between adjacent levels of the same L. The three models repesent different assumptions about the shape of the nuclear charge distribution. They are I: sharp cutoff, radius of 6.545 f; II: Woods-Saxon shape with radius of 5.754 f and diffusness of 0.65 f; III: as II with radius 5.989 f and diffusness of 0.55 f. All three models have the same rms radius and total charge of 82 protons. The tau mass has been taken to be twice the average nucleon mass and center of mass corrections have not been made.

Table II: Transition energies in Mev as indicated by model described in Table I. The near equivelance of nP-nS and nD-nP, where n is the radial quantum number, only for n=1 is a measure off the limits of the harmonic spectrum independent of the nuclear surface.

## FIGURE CAPTION

Fig. 1: Sketch of the tau- atomic orbits superimposed upon the sharp cut off nuclear model (Model I). Labels refer to Table I.

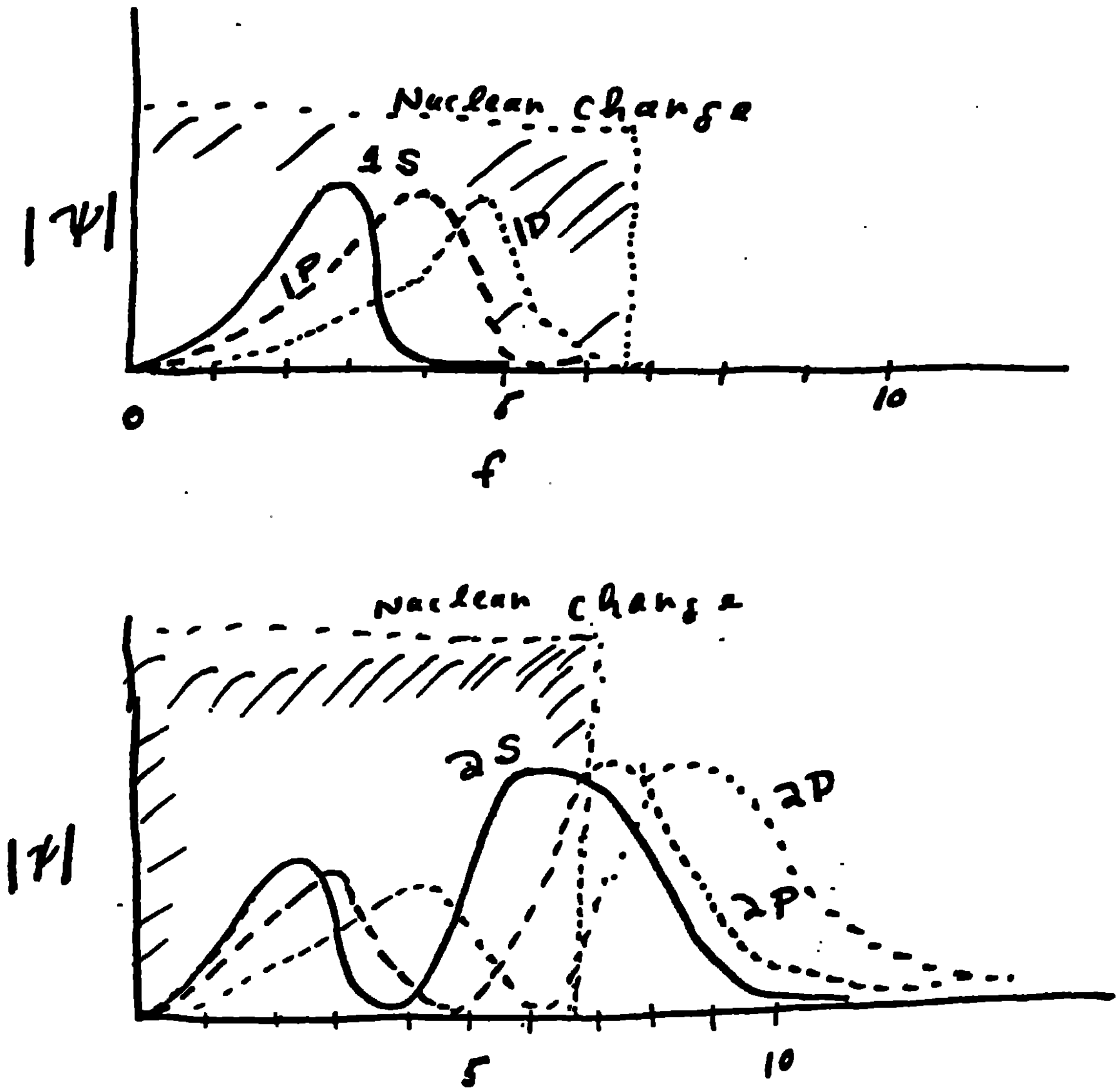


FIGURE 1

TABLE I - MODEL I

THESE ARE FROM SHARP CUTOFF MODEL

| L | n | e | space | max wf | delta |
|---|---|---|---|---|---|
| 0 | 1 | -22.63 | 16 | 2.6 | |
| | | | | | 5.804 |
| | 2 | -16.826 | 16 | 5.4 | |
| | | | | | 4.94 |
| | 3 | -11.886 | 24 | 8 | |
| | | | | | 3.237 |
| | 4 | 8.649 | 24 | 11.3 | |
| | | | | | 2.143 |
| | 5 | -6.506 | 40 | 15.2 | |
| | | | | | 1.4572 |
| | 6 | -5.0488 | 50 | 20 | |
| 1 | 1 | -19.693 | 24 | 3.84 | |
| | | | | | 5.583 |
| | 2 | -14.11 | 30 | 6.6 | |
| | | | | | 4.087 |
| | 3 | -10.023 | 36 | 9.36 | |
| | | | | | 2.627 |
| | 4 | -7.396 | 36 | 12.96 | |
| | | | | | 1.741 |
| | 5 | -5.655 | 36 | 17.28 | |
| | | | | | 1.205 |
| | 6 | -4.45 | 50 | 22.5 | |
| 2 | 1 | -16.781 | 20 | 4.6 | |
| | | | | | 5.098 |
| | 2 | -11.683 | 24 | 7.44 | |
| | | | | | 3.246 |
| | 3 | -8.437 | 30 | 11.1 | |
| | | | | | 2.106 |
| | 4 | -6.331 | 30 | 15 | |
| | | | | | 1.418 |
| | 5 | -4.913 | 40 | 19.6 | |
| | | | | | 0.9953 |
| | 6 | -3.9177 | 60 | 25.2 | |
| | | | | | 0.7217 |
| | 7 | -3.196 | 60 | 31.2 | |

TABLE I - MODEL II

FROM WOODS SAXON SHAPE
Ro = 5.754, a = 0.65

| L | n | e | space | max wf | delta |
|---|---|---|---|---|---|
| 0 | 1 | 23.4555 | 12 | 2.52 | |
| | | | | | 6.3374 |
| | 2 | 17.1181 | 24 | 5.28 | |
| | | | | | 5.0241 |
| | 3 | -12.094 | 24 | 7.92 | |
| | | | | | 3.3294 |
| | 4 | 8.7646 | 36 | 11.16 | |
| | | | | | 2.1845 |
| | 5 | -6.5601 | 50 | 15 | |
| | | | | | 1.4812 |
| | 6 | -5.0989 | 50 | 19.5 | |
| 1 | 1 | -20.1635 | 24 | 3.84 | |
| | | | | | 5.855 |
| | 2 | -14.3085 | 24 | 6.72 | |
| | | | | | 4.1446 |
| | 3 | -10.1639 | 36 | 10.08 | |
| | | | | | 2.6768 |
| | 4 | -7.4871 | 36 | 13.68 | |
| | | | | | 1.774 |
| | 5 | -5.7131 | 50 | 18 | |
| | | | | | 1.2217 |
| | 6 | -4.4914 | 50 | 23 | |
| 2 | 1 | -16.9743 | 20 | 4.8 | |
| | | | | | 5.1435 |
| | 2 | -11.8308 | 30 | 8.1 | |
| | | | | | 3.3116 |
| | 3 | -8.5192 | 30 | 11.4 | |
| | | | | | 2.1334 |
| | 4 | -6.3858 | 40 | 15.6 | |
| | | | | | 1.4336 |
| | 5 | -4.9522 | 50 | 20.5 | |
| | | | | | 1.0026 |
| | 6 | -3.9496 | 60 | 25.8 | |

TABLE I - MODEL III

FROM WOODS SAXON SHAPE
Ro = 5.989, a = 0.55

| L | n | e | space | max wf | delta |
|---|---|---|---|---|---|
| 0 | 1 | 23.215 | 12 | 2.52 | |
| | | | | | 6.1862 |
| | 2 | 17.0288 | 24 | 5.28 | |
| | | | | | 4.9924 |
| | 3 | -12.0364 | 24 | 7.92 | |
| | | | | | 3.3057 |
| | 4 | 8.7307 | 36 | 11.16 | |
| | | | | | 2.1739 |
| | 5 | -6.5568 | 50 | 15 | |
| | | | | | 1.4728 |
| | 6 | -5.084 | 50 | 19.5 | |
| 1 | 1 | -20.0287 | 24 | 3.84 | |
| | | | | | 5.7793 |
| | 2 | -14.2494 | 24 | 6.72 | |
| | | | | | 4.1256 |
| | 3 | -10.1236 | 36 | 10.08 | |
| | | | | | 2.6618 |
| | 4 | -7.4618 | 36 | 13.68 | |
| | | | | | 1.7658 |
| | 5 | -5.696 | 50 | 18 | |
| | | | | | 1.2167 |
| | 6 | -4.4793 | 50 | 23 | |
| 2 | 1 | -16.917 | 20 | 4.8 | |
| | | | | | 5.1264 |
| | 2 | -11.7906 | 30 | 8.1 | |
| | | | | | 3.2959 |
| | 3 | -8.4947 | 30 | 11.4 | |
| | | | | | 2.125 |
| | 4 | -6.3697 | 40 | 15.6 | |
| | | | | | 1.4291 |
| | 5 | -4.9406 | 50 | 20.5 | |
| | | | | | 1.0017 |
| | 6 | -3.9389 | 60 | 25.8 | |

TABLE II

MODEL

| TRANSITION | I | II | III |
| --- | --- | --- | --- |
| 1p-1s | 2.9370 | 3.2920 | 3.1863 |
| 1d-1p | 2.910 | 3.1892 | 3.1117 |
| 2p-2s | 2.7160 | 2.8096 | 2.7794 |
| 2d-2p | 2.4270 | 2.4777 | 2.4588 |
| 3p-3s | 1.8630 | 1.9301 | 1.9128 |
| 3d-3p | 1.5860 | 1.6447 | 1.6289 |
| 4p-4s | 1.2530 | 1.2775 | 1.2689 |
| 4d-4p | 1.0650 | 1.1013 | 1.0921 |
| 5p-5s | 0.8510 | 0.8670 | 0.8608 |
| 5d-5p | 0.7420 | 0.7609 | 0.7554 |
| 6s-6p | 0.5988 | 0.6075 | 0.6047 |
| 6d-dp | 0.5323 | 0.5418 | 0.5404 |